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Effect of Mass Transfer And Hall Current On Unsteady Mhd Flow Of A Viscoelastic Fluid In A Porous Medium.

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Abstract— *The paper investigated the effect of mass transfer and Hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium. The resultant equations have been solved analytically. The velocity, temperature and concentration distributions are derived, and their profiles for various physical parameters are shown through graphs. The coefficient of Skin friction, Nusselt number and Sherwood number at the plate are derived and their numerical values for various physical parameters are presented through tables. The influence of various parameters such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscoelasticity parameter, Hartmann number, Hall parameter, and the frequency of oscillation on the flow field are discussed. It is seen that, the velocity increases with the increase in Gc, Gr, M, m and K, and it decreases with increase in Sc, n and Pr, temperature decreases with increase in Pr and n, Also, the concentration decreases with the increase in Sc and n.*

Key Words— *Viscoelastic, Hall Current, Magnetohydrodynamics, Porous medium.*

I. INTRODUCTION

Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. It is important in the design of MHD generators and accelerators in geophysics, underground water storage system, soil sciences, astrophysics, nuclear power reactor, solar structures, and so on.

Moreover, there exist flows which are caused not only by temperature differences but also by concentration differences. There are several engineering situations wherein combined heat and mass transport arise such as dehumidifiers, humidifiers, desert coolers, and chemical reactors etc. the interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors – notable amongst them are Singh and Singh (2012) they investigated MHD flow of Viscous Dissipation and Chemical Reaction over a Stretching porous plate in a porous medium numerically. Hall Effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with Heat Source/Sink was studied by Sharma *et al.* (2007).

Das and Jana (2010) examined Heat and Mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. The effects of chemical reaction, Hall current and Ion – Slip currents on MHD micropolar fluid flow with thermal Diffusivity using Novel Numerical Technique was studied by Motsa and Shateyi (2012). Aboeldahab and Elbarby (2001) examined Hall current effect on Magnetohydrodynamics free convection flow past a Semi – infinite vertical plate with mass transfer. Hall current effect with simultaneous thermal and mass diffusion on unsteady hydromagnetic flow near an accelerated vertical plate was investigated Acharya *et al.* (2001). Takhar (2006) studied Unsteady flow free convective flow over an infinite vertical porous platedue to the combined effects of thermal and mass diffusion, magnetic field and Hall current.

Exact Solution of MHD free convection flow and Mass Transfer near a moving vertical porous plate in the presence of thermal radiation was investigated by Das (2010). Anjali Devi and Ganga examined effects of viscous and Joules dissipation and MHD flow, heat and mass transfer past a Stretching porous surface embedded in a porous medium. Sonth *et al.* (2012) studied Heat and Mass transfer in a viscoelastic fluid over an accelerated surface with heat source/sink and viscous dissipation. Shateyi *et al.* (2010) investigated The effects of thermal Radiation, Hall currents, Soret and Dufour on MHD flow by mixed convection over vertical surface in porous medium. Effect of Hall currents and Chemical reaction and Hydromagnetic flow of a stretching vertical surface with internal heat generation / absorption was examined by Salem and El-Aziz (2008)..

II. MATHEMATICAL FORMULATION

We consider the unsteady flow of a viscous incompressible and electrically conducting viscoelastic fluid over an infinite porous plate with oscillating temperature and mass transfer. The x- axis is assumed to be oriented vertically upwards along the plate and the y-axis is taken normal to the plane of the plate. It is assumed that the plate is electrically non – conducting and a uniform magnetic field of straight B_0 is applied normal to the plate. The induced magnetic field is assumed constant. So that $\vec{B}=\left(0,B_0,0\right)$,. The plate is subjected to a constant suction velocity.

1. The equation of conservation of charge $\nabla \cdot J=0$, gives constant.
2. $$\vec{J}=\omega_e\tau_e(\vec{J}\times\vec{E})=\sigma\left(\vec{V}\times\vec{B}+\frac{\nabla P_e}{en_e}\right) \tag{1}$$

Equation (1) reduces to

$$\left.\begin{aligned} J_{x^*} &= \frac{\sigma B_0}{(1+m^2)}\left(mu^*-\omega^*\right) \\ J_{y^*} &= \frac{\sigma B_0}{(1+m^2)}\left(u^*-m\omega^*\right) \end{aligned}\right\} \tag{2}$$

Where

$m=\omega_e\tau_e$

is the Hall parameter.

The governing equations for the momentum, energy and concentration are as follows;

$$\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}=\nu\frac{\partial u}{\partial y}-k_1\frac{\partial^3 u}{\partial y^2\partial t}-\sigma B_0{}^2\frac{(u+m\omega)}{\rho(1+m^2)}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\nu u}{k^*} \tag{3}$$

$$\frac{\partial \omega}{\partial t}+v_0\frac{\partial \omega}{\partial y}=\nu\frac{\partial^2 \omega}{\partial y^2}-k_1\frac{\partial^3 \omega}{\partial y^2\partial t}-\sigma B_0{}^2\frac{(\omega-mu)}{\rho(1+m^2)}-\frac{u\omega}{k^*} \tag{4}$$

$$\frac{\partial T}{\partial t}+v_0\frac{\partial T}{\partial y}=\frac{K_T}{\rho Cp}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$\frac{\partial C}{\partial t}+v_0\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{6}$$

The boundary conditions of the problem are:

$$\begin{aligned} &u=0,\omega=0,T=T_\infty+(T_\omega-T_\infty)e^{int},C=C_\infty+(C_\omega-C_\infty)e^{int} \text{ at } y=0 \\ &u\to 0,\omega\to 0,T\to 0,C\to 0 \text{ as } y\to\infty \end{aligned} \tag{7}$$

Where u and v are the components of velocity in the x and y direction respectively, g is the acceleration due to gravity, β and β^* are the coefficient of volume expansion, K is the kinematic viscoelasticity, ρ is the density, μ is the viscosity, ν is the kinematic viscosity, K_T is the thermal conductivity, Cp is the specific heat in the fluid at constant pressure, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, D is the molecular diffusivity, T_ω is the temperature of the plane and T_∞ is the temperature of the fluid far away from plane. C_ω is the concentration of the plane and C_∞ is the concentration of the fluid far away from the plane.

And $v=-v_0$, the negative sign indicate that the suction is towards the plane.

Introducing the following non-dimensionless parameters

$$\left.\begin{aligned}&\eta=\frac{v_0 y}{v},t=\frac{{v_0}^2 t}{4v},u=\frac{u}{v_0},\omega=\frac{\omega^*}{v_0},\theta=\frac{T-T_\infty}{T_\omega-T_\infty},C=\frac{C-C_\infty}{C_\omega-C_\infty}\\&Gr=\frac{g\beta v(T_\omega-T_\infty)}{{v_0}^2},Gc=\frac{g\beta c v(C_\omega-C_\infty)}{{v_0}^2},M=\frac{\sigma {B_0}^2 v}{\rho {v_0}^2},\Pr=\frac{\mu Cp}{K_T}\\&Sc=\frac{v}{D},K=\frac{k_1 {v_0}^2}{4v^2},k=\frac{k^* {v_0}^2}{v^2}\end{aligned}\right\} \quad (8)$$

Substituting the dimensionless variables in (8) into (3) to (6), we get (dropping the bars)

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial \eta}=\frac{\partial^2 u}{\partial \eta^2}-\frac{K\partial^3 u}{4\partial\eta^2\partial t}-\frac{M(u+m\omega)}{(1+m^2)}-\frac{u}{k}+Gr\theta+GcC \quad (9)$$

$$\frac{1}{4}\frac{\partial \omega}{\partial t}-\frac{\partial \omega}{\partial \eta}=\frac{\partial^2 \omega}{\partial \eta^2}-\frac{K\partial^3 \omega}{4\partial\eta^2\partial t}-\frac{M(\omega-mu)}{(1+m^2)}-\frac{u}{k} \quad (10)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial \eta}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} \quad (11)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-\frac{\partial C}{\partial \eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2} \quad (12)$$

The corresponding boundary conditions are

$$\begin{aligned}&u(0,t)=0,\omega(0,t)=0,\theta(0,t)=e^{int},C(0,t)e^{int}\ \text{at y}=0\\&u(\infty,t)=\omega(\infty,t)=\theta(\infty,t)=C(\infty,t)=0\ \text{as y}\to\infty\end{aligned} \quad (13)$$

Equations (9) and (10) can be combined into a single equation by introducing the complex velocity.

$$U=u(\eta,t)+i\omega(\eta,t) \quad (14)$$

Where

$i=\sqrt{-1}$

Thus,

$$\frac{1}{4}\frac{\partial U}{\partial t}-\frac{\partial U}{\partial \eta}=\frac{\partial^2 U}{\partial \eta^2}-\frac{K\partial^3 U}{4\partial\eta^2\partial t}-\frac{M(1-im)U}{(1+m^2)}-\frac{U}{k}+Gr\theta+GcC \quad (15)$$

With boundary conditions:

$$\begin{aligned}&U(0,t)=0,\theta(0,t)=e^{i\Omega t},C(0,t)e^{i\Omega t}\ \text{at}\ \eta=0\\&U(\infty,t)=\theta(\infty,t)=C(\infty,t)\to 0\ \text{as}\ \eta\to\infty\end{aligned} \quad (16)$$

where Gr, is the thermal Grashof number, Gc, is the mass Grashof number, Sc, is the Schmidt number, Pr, is the Prandtl number, K, is the viscoelastic Parameter, M, is the Hartmann number and k, is the pearmeability.

III. METHOD OF SOLUTIONS

To solve (11), (12) and (15) subject to the boundary conditions (16), we assume solutions of the form

$$U(\eta,t)=U_1(\eta)e^{int} \quad (17)$$

$$\theta(\eta,t)=\theta_1(\eta)e^{int} \quad (18)$$

$$C(\eta,t)=C_1(\eta)e^{int} \quad (19)$$

where $U_1(\eta)$ $\theta_1(\eta)$ and $C_1(\eta)$ are to be determined.

Substituting (17) to (19) into (11), (12) and (15), Comparing harmonic and non harmonic terms, we obtain

$$U'' + \frac{U'}{P_1} - P_3 U = -\frac{Gr\theta}{4P_1 e^{int}} - \frac{GcC}{4P_1 e^{int}} \tag{20}$$

$$\theta_1'' + \Pr \theta_1' - \frac{1}{4} in\theta_1 = 0 \tag{21}$$

$$C_1 + ScC_1 - \frac{1}{4} inC_1 = 0 \tag{22}$$

and boundary conditions give

$$\left.\begin{array}{l} U_1(0) = \theta_1(0) = C_1(0) = 1 \text{ at } \eta = 0 \\ U_1(\infty) \to 0, \theta_1(\infty) \to 0, C_1(\infty) \to 0 \text{ as } \eta \to \infty \end{array}\right\} \tag{23}$$

where the primes represents differentiation with respect to η.

Solving (20) to (22) subject to the boundary conditions (23) and (25). And substituting the obtained solutions into (17) to (19) respectively. Then the velocity field can be expressed as

$$U(\eta, t) = \left[A_6 e^{-m_6 n} + A_7 e^{-m_2 y} + A_8 e^{-m_4 y} \right] e^{\text{int}} \tag{24}$$

And, the temperature field is given by

$$\theta(\eta, t) = e^{-m_2 \eta} \cdot e^{\text{int}} \tag{25}$$

Similarly, the concentration distribution gives

$$C(\eta, t) = e^{-m_4 \eta} \cdot e^{\text{int}} \tag{26}$$

The Skin friction, Nusselt number and Sherwood number is obtained by differentiating (24) to (26) and at evaluated at $\eta = 0$ respectively.

$$-\left.\frac{\partial U(\eta, t)}{\partial \eta}\right|_{\eta=0} = \left[m_6 A_6 + m_2 A_7 + m_4 A_8 \right] e^{\text{int}} \tag{27}$$

$$-\left.\frac{\partial \theta(\eta, t)}{\partial \eta}\right|_{\eta=0} = m_2 e^{\text{int}} \tag{28}$$

$$-\left.\frac{\partial C(\eta, t)}{\partial \eta}\right|_{\eta=0} = m_4 e^{\text{int}} \tag{29}$$

IV. RESULTS AND DISCUSSION

The effect of mass transfer and Hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium has been formulated and solved analytically. In order to understand the flow of the fluid, computations are performed for different parameters such as Gr, Gc, Sc, Pr, n, M, K, and m.

4.1 V**elocity profiles**

Figures 1-8 represent the velocity profiles, figures 9 and 10 depict the temperature profiles and figures 11 and 12 show the concentration profiles with varying parameters respectively.

The effect of velocity for different values of (Pr = 0.025, 0.71, 1, 3) is presented in figure 1, the graph show that velocity decreases with increase in Pr.

The effect of velocity for different values of (Sc = 0.3, 0.6, 0.8, 1, 2.01) is given in figure 2, the graph show that velocity decreases with the increase in Sc.

Figure 3 denotes the effect of velocity for different values of (n =1, 2, 3, 4, 7), it is seen that velocity increases with the increase in n.

The effect of velocity for different values of (m = 1, 2, 3, 4, 7) is shown in figure 4, it depict that velocity increases with increase in m.

Figure 5 depicts the effect of velocity for (M = 1, 2, 3, 4, 7), the graph show that velocity increases with the increase in M.

The effect of velocity for different values of (Gr = 1, 2, 3, 4, 7) is presented in figure 6, it is seen that velocity increases with the increase in Gr.

The effect of velocity for different values of (Gc = 1, 2, 3, 4, 7) is displayed in figure 7, it is observed that velocity increases with the increase in Gc.

The effect of velocity for different values of (K = 0.0001, 0.01, 0.05, 0.08, 0.1) is shown in figure 8, it is observed that velocity increases with the increase in K.

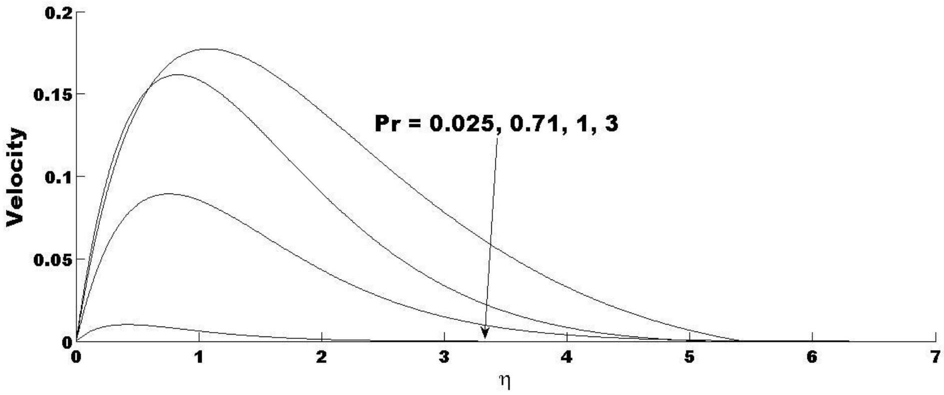

Figure 1. Velocity profiles for different values of Pr.

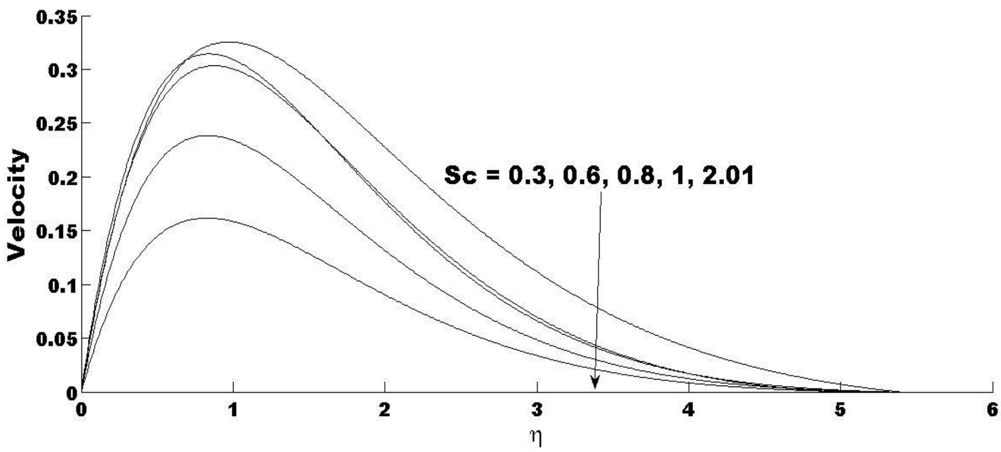

Figure 2. Velocity profiles for different values of Sc.

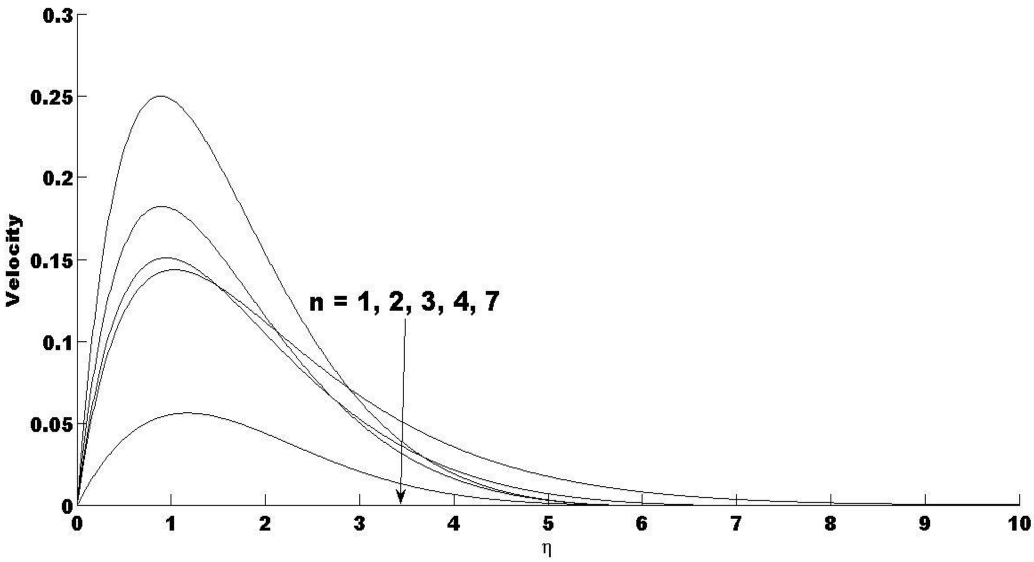

Figure 3. Velocity profiles for different values of n.

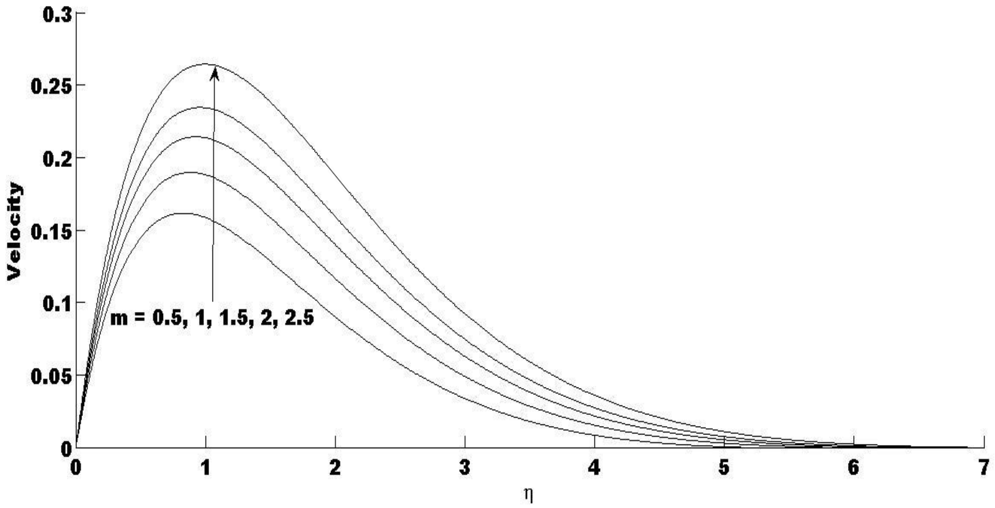

Figure 4. Velocity profiles for different values of m.

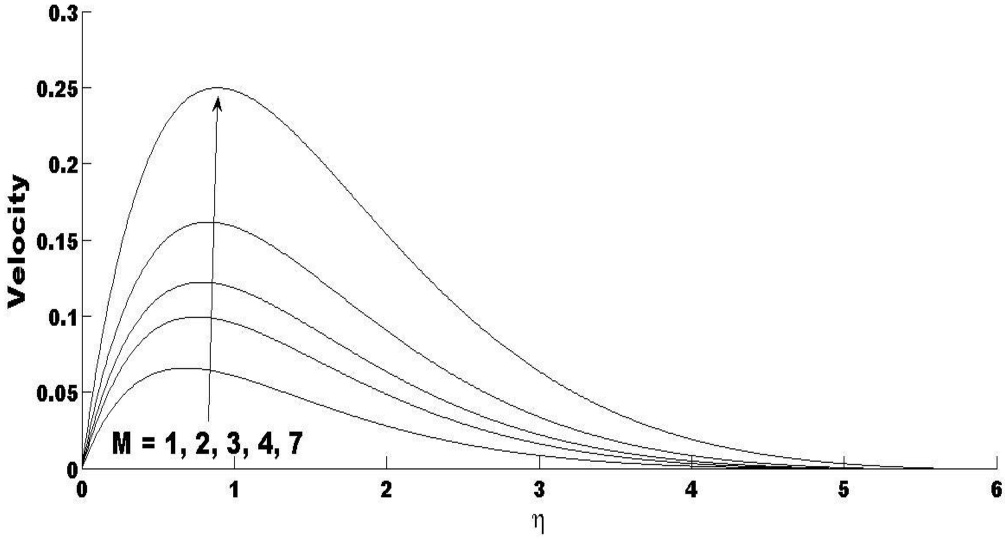

Figure 5. Velocity profiles for different values of M.

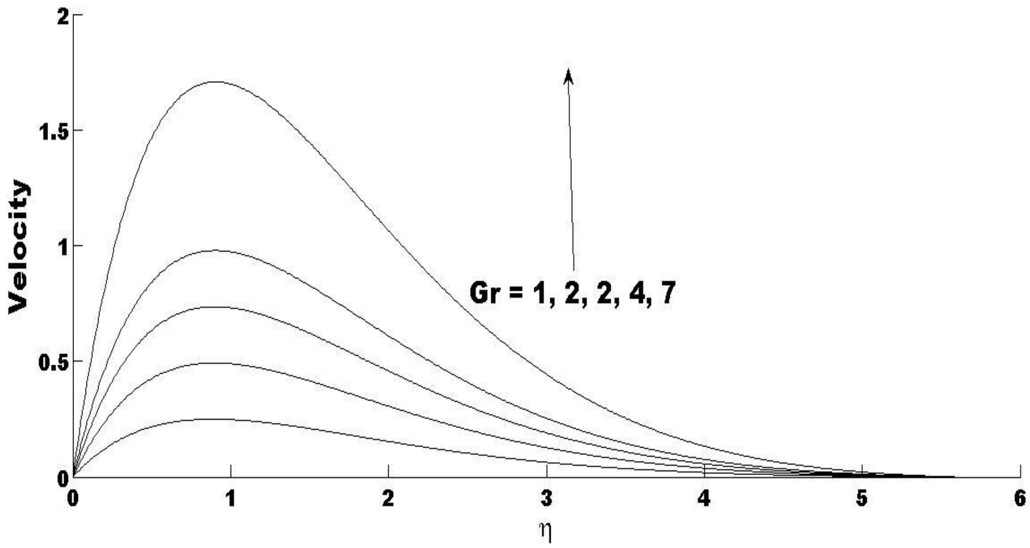

Figure 6. Velocity profiles for different values of Gr.

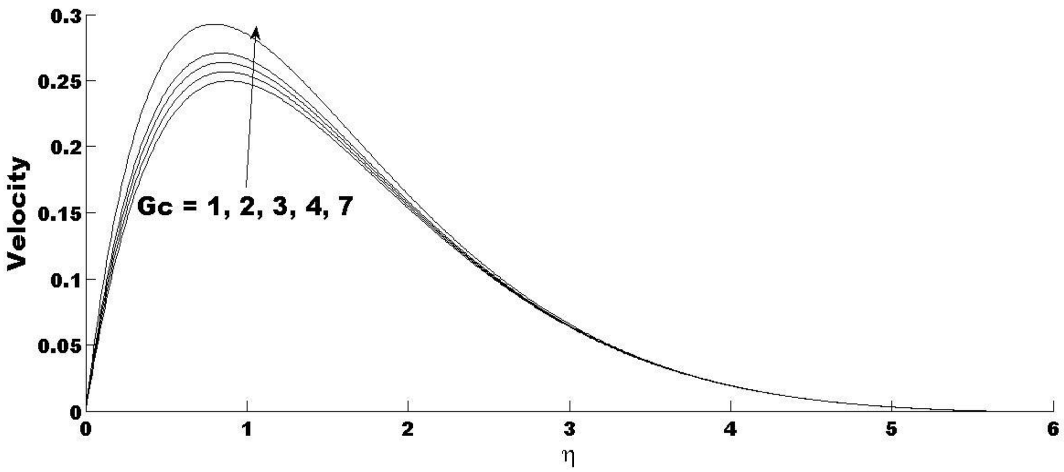

Figure 7. Velocity profiles for different values of Gc.

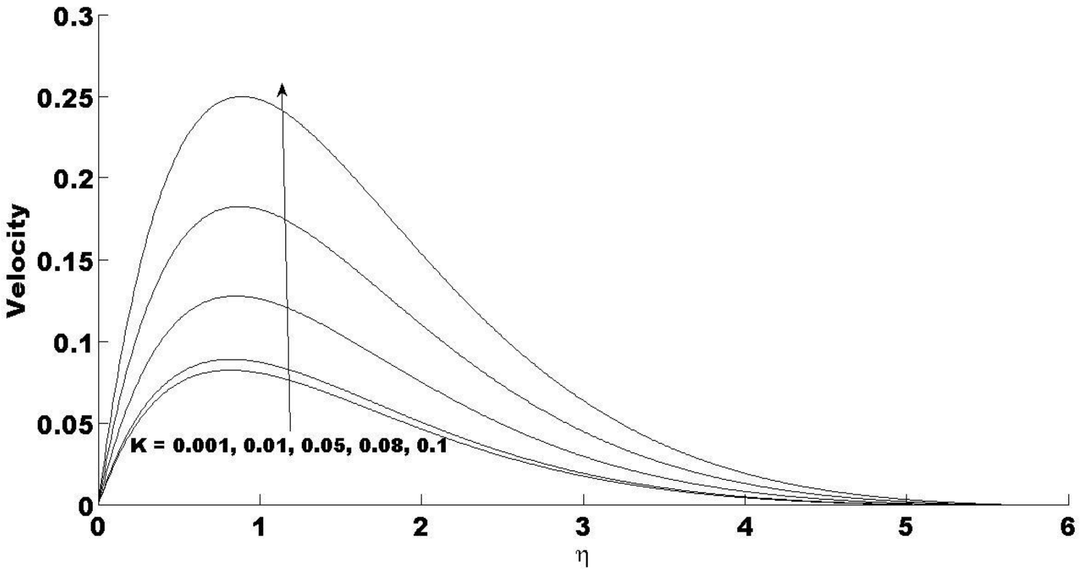

Figure 8. Velocity profiles for different values of K.

4.2 Temperature profiles

In figure 9, the effect of temperature for different values of (Pr = 0.025, 0.71, 1, 3, 7) is given. The graph show that temperature increases with increasing Pr. Figure 10 shows the effect of temperature for different values of (n = 0.025, 0.71, 1, 3, 7). The graph show that temperature decreases with decreasing n.

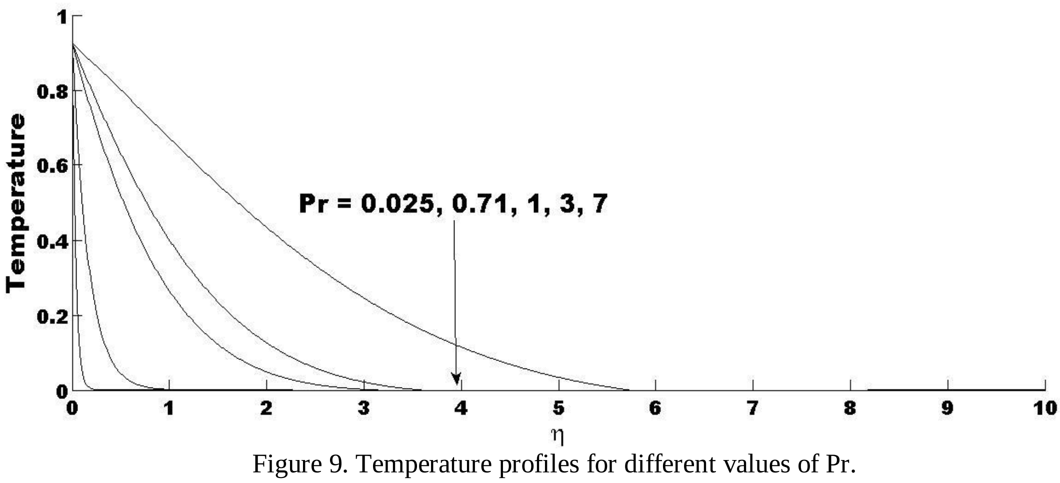

Figure 9. Temperature profiles for different values of Pr.

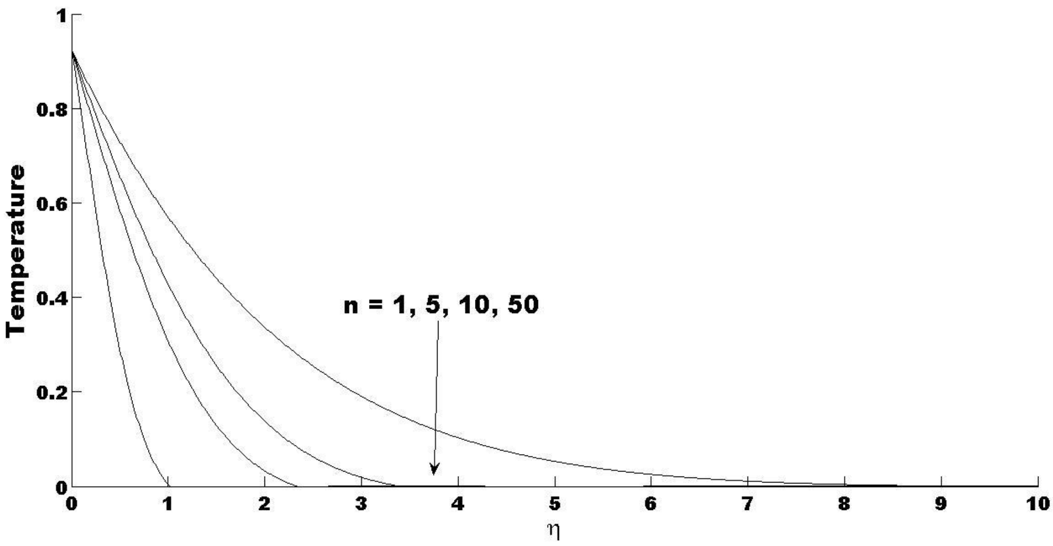

Figure 10. Temperature profiles for different values of n.

4.3 Concentration profiles

Figure 11 depicts the effect of concentration for (Sc = 0.3, 0.6, 0.8, 1, 2.01), it is seen that concentration increases with the increase in Sc. The effect of concentration for (n = 0.3, 0.6, 0.8, 1, 2.01) is given in figure 12, it is seen that concentration decreases with the decrease in n.

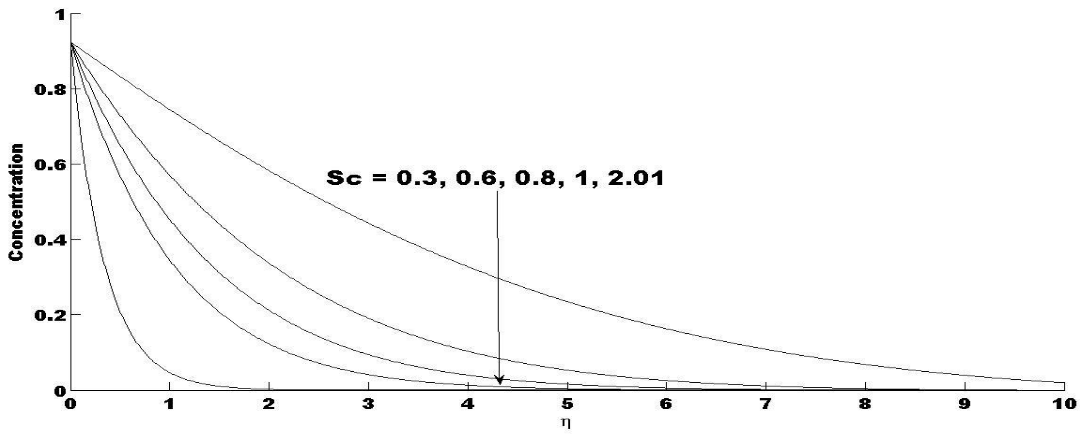

Figure 11. Concentration profiles for different values of Sc.

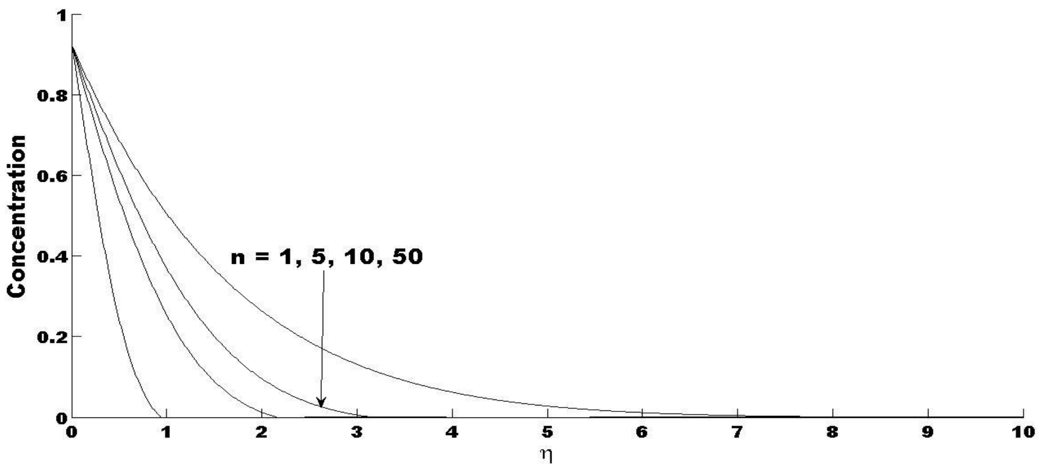

Figure 12. Concentration profiles for different values of n.

Table 1 to 3 represent the Skin friction, Nusselt number and Sherwood number respectively.
Table 1 depicts that the Skin friction increases with increase in K, Gc, Gr, M and m, and decreases with increase in Sc, n and Pr.
Table 2 represents that the rate of heat transfer decreases with increase in Pr and n.
Table 3 shows that the Sherwood number increases with increase in n and Sc.

Table 1: Skin friction τ

n	Gc	Sc	K	Gr	M	Pr	M	τ
1	1	0.3	0.1	1	1	0.71	0.5	4.1082
3	1	0.3	0.1	1	1	0.71	0.5	3.5219
1	1	0.6	0.1	1	1	0.71	0.5	3.7548
1	4	0.3	0.1	1	1	0.71	0.5	4.5671
1	1	0.3	1	1	1	0.71	0.5	4.3654
1	1	0.3	0.1	4	1	0.71	0.5	4.8760
1	1	0.3	0.1	1	3	3	0.5	4.9006
1	1	0.3	0.1	1	1	0.71	2	4.0126
1	1	0.3	0.1	1	1	0.71	0.5	4.8741

Table 2: Nusselt number

N	Pr	Nu
1	0.71	0.7181
4	0.71	0.6400
1	3	0.6830

Table 3: Sherwood number

n	Sc	Sh
1	0.3	0.4015
5	0.3	0.3890
1	0.6	0.3977

V. SUMMARY AND CONCLUSION

We have examined and solved analytically the governing equations for the effect of mass transfer and Hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium analytically. In order to point out the effect of physical parameters namely; thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscoelasticity parameter, Hartmann number, Hall parameter and the frequency of oscillation on the flow field. We observe that, the velocity increases with the increase in Gc, Gr, M, m, and K, and it decreases with increase in Sc, n, and Pr. Temperature decreases with increase in Pr and n, and concentration decreases with the increase in Sc and n. The Skin friction increases with increase in Gc, Gr, M, m and K, and decreases with increase in Sc, n, and Pr. The rate of heat transfer decreases with increase in Pr and n, The Sherwood number decreases with the increase in Sc and n.

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APPENDIX

$$a=\sqrt{\frac{\Pr^2+in\Pr}{4}}\ b=\sqrt{\frac{Sc^2+inSc}{4}}\ m_2=\frac{\Pr}{2}+a\ P_1=\left(1-\frac{inK}{4}\right)\ P_2=\left(\frac{in}{4}+\frac{1}{k}+\frac{M\left(1-im\right)}{4\left(1+m^2\right)}\right)$$

$$P_3=\frac{P_2}{P_1}$$

$$m_4=\frac{Sc}{2}+b\ A_7=-\frac{Gr}{P_1\left(m_2^2-\frac{m_2}{P_1}-P_3\right)}\ A_8=-\frac{Gc}{P_1\left(m_4^2-\frac{m_4}{P_1}-P_3\right)}\ A_6=-A_7-A_8$$